## Bruce and Alice's Adventures in Ruritania - by Zoltan P. Dienes <br> CHAPTER VI.

## Bruce and Alice go on holiday in EIGHTLAND.

Everyone was very excited about the prospect of going to EIGHTLAND. They were planning to stay several days in EIGHTLAND, as there was still some time left out of the children's week's holiday from school. There was a ferry service from Twoport to Eightsome-on-Sea, which was a seaside resort as well as an important port in EIGHTLAND. The crossing only took about an hour (not an AWA!), but you had to travel to Twoport first which was about 11000000000 TWOLAND decimas away. This was not very far, Mr. Koro reckoned it was about two zero two THREELAND decimas. Bruce made a quick calculation in his head and worked it out that it must be about twelve kilometers.

Bruce and Alice had by now worked out the various distances in TWOLAND as well as in THREELAND measures, and they made up some tables so they could easily transform different decimas into each other as well as transform them into kilometers or meters.

The table looked like this

| TWOLAND | Metric | THREELAND | Metric |
| :--- | :--- | :--- | :--- |
| una | 3 cm | una | 3 cm |
| dua | 6 cm | dua | 9 cm |
| tria | 12 cm | tria | 27 cm |
| quarta | 24 cm | quarta | 81 cm |
| quinta | 48 cm | quinta | 2.43 m |
| sexta | 96 cm | sexta | 7.29 m |
| septima | 1.92 m | septima | 21.87 m |
| octava | 3.84 m | octava | 65.61 m |
| nona | 7.68 m | nona | 196.83 m |
| decima | 15.36 m | decima | 590.49 m |

So about 1100000000 TWOLAND decimas $=$ (approx) 12 km ,
and 202 THREELAND decimas $=$ (approx) 12 km .

If you do not think these figures are right, you can always check them.
They soon piled into the Kotos' car and Mr. Koto took the wheel. They passed the border into TWOLAND and they were on their way to Twoport!

What's EIGHTLAND tennis like?", asked Bruce as they were travelling along the road, just to make conversation.
"Perhaps Alo will tell you", said Mr. Koto, who thought he had better keep his eyes on the road.
"I'll tell you", agreed Alo, "You can cut a square in half in four different ways, by folding one half over the other half. You can draw lines along the folds and that will make eight triangles. Look, I'll draw them for you." and he took a piece of paper out of his pocket and drew this diagram:

"Each of the triangles is used as a court, and of course eight people play, one in each court. Their courts are much bigger than ours, so as to give room for the players to move about. I hope we shall have a chance to have a game, but we find the rules rather complicated, and Threelanders don't usually do very well. People from TWOLAND do better, as they are used to doing things in twos and fours and eights. You see, eight is expressed as one zero zero zero in TWOLAND and so for them it is a round number. In our land eight is not a round number, we call it two two! So we make a lot of mistakes in EIGHTLAND tennis and nearly always lose."
"In FOURLAND there are two kinds of tennis, with different sets of rules", said Bruce, " How many different sets of rules are there in EIGHTLAND for playing tennis?"
"There are five different kinds", replied Alo, "I think I can only play one of those at all reasonably. In EIGHTLAND schools they practice tennis a lot, as tennis there is a part of the mathematics curriculum. In our schools you would eventually learn to play NINELAND tennis, which is not against the law in our land. We don't make quite such a fuss about mathematics as they do in EIGHTLAND. In EIGHTLAND you really have to be good at mathematics or you don't get on at all well! You see, we are really rather lucky, because our number is a prime number but eight is not, and you can do less things with a prime number than a complicated number like eight, which is two times two times two!

Oh, look!", interrupted Alice, "I can see the sea! We are coming down to the bay! Isn't the sea lovely and blue!"
"Yes, it's very lovely", agreed all the children "and there is Twoport, and there is the boat we are going to sail in!"

They followed the winding road down to the coast and then along the shore till they entered the little town of Twoport. There was indeed a little harbour there with a few sailboats anchored along the quay side. One larger looking boat was tied up alongside the pier, which was the boat that would take them to

Eightsome-on-Sea. Mr. Koto drove on to the pier and enquired whether there was room for the car on the boat.
"Yes, there is plenty of room", said one port official, "there is nobody else booked to go over with a car today"

So after everyone had got out of the car, Mr. Koto carefully drove the car on to the deck of the ferryboat, carefully negotiating the one zero planks that had been placed there for that purpose. They soon made themselves comfortable in the boat's lounge. After about one zero one minutes the whistle was sounded one one times, which was the signal for departure. The men on the pier let go of the ropes, which were pulled in by the ship's sailors, the engineer started the propeller and the boat quietly slid out of the bay into the open sea. The children loved the calm blue sea. The water was so clear that you could see the sea bottom the whole time, all the way across. There were lots of corals, many sea plants, that waved about in the currents and they could see a large number of fish which all had the most exotic colors. Bruce and Alice were not used to such calm and clear waters, so it was hard to tear them away from the rail, where they stood, looking down into the sea, marveling at all the beautiful creatures and plant life that they were passing.

But finally Mrs. Koto was able to entice them away with some delicious sandwiches and fruit juices which she had prepared for the journey. The time soon passed and before they knew it, they were docking at Eightsome-on-Sea. This was indeed a very pretty place, with a promenade along the waterfront, with palm trees and fountains playing in the surrounding gardens. It was all beautifully kept. It was not long before they had the car off the ferry and drove off along the promenade.
"Don't forget, children", said Mrs. Koto, "that now we are in EIGHTLAND, we must obey their rules. As a matter of fact, $I$ don't intend to go very far just yet, as there are one or two things I would like to show you right in this area."

They drove along for a little while until they came to a building surrounded by large gardens.
"What's that building?" asked Alice.
"That's the children's prison", said Mr. Koto.
"Good heavens!", cried Alice "Surely that is not a prison. It looks such a nice place! Look at the lovely gardens and the pleasant windows with pretty curtains! And there are some children playing outside, is it really a prison, or are you teasing us?"
"It really is a prison", said Mr. Koto "I know what you are thinking. Your prisons are rather different from ours, aren't they Bruce? Anyway, you don't have prisons for children, do you?"
"No, we do not", said Bruce and Alice together. They were just about to say that they were much too civilized to send children to prison, but they just stopped themselves in time, as they realized that people did things differently in different places. Who was to say who was right?
"But why is the prison such a pleasant place?", asked Alice.
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#### Abstract

"Well, why should it be a nasty place?" retorted Mr. Koto, "I know, your prisons in your country are made particularly unpleasant. Your idea is that by making them so unpleasant, people would not want to go there, and so they would not do the things for which you would want to send them to prison!" "That's right", said Bruce, "we send people to prison who do wicked things, and we hope they will get discouraged from doing wicked things, in case they might be sent to prison!"


"Yes, I understand this", said Mr. Koto, "In our country we don't look at it that way. We feel that if people have done something that isn't right, they should learn how to do it right. If we treat them badly, surely they won't learn! So when we send people to prison, whether they are adults or children, we make sure they learn not only what they are supposed to learn, but we also make sure that they have a good time. Then they will want to do things right, because they will think of the good time they had while they were learning how to do things right "
"That's a very strange idea of a prison" said Alice, "I suppose it works. Does it?"
"Well", said Mr. Koto, "It does work, because we only have very few people in prison, there are always vacant rooms"
"You call them rooms?", asked Bruce incredulously "we call them cells in our country, and the prisoners are locked inside!"
"Yes, we call them rooms", said Mrs. Koto "and they are pleasantly furnished with everything you need"
"I must tell them about all this at home!", cried Bruce "They'll never believe us!"
"There is always a table where you can work", said Mrs. Koto,"and a library with books where you can look things up, there are also some workshops for learning some of the manual skills people might need. They all learn what they came to learn."
"Do the children like it in prison?", asked Alice.
"Well, they don't exactly look forward to going", said Mrs. Koto, "because they usually prefer to be at home, although it is not unpleasant for them in prison. They do look forward to going home when they have finished their time."

By this time Mr. Koto had driven through the prison gates and pulled up at the main entrance. The superintendent of the prison was there to greet them.
"Good morning Mrs. Koto, good morning Mr. Koto!", said the superintendent, "I see you have brought your whole family, oh, and some more children! Who are these children?"
"Let me introduce them to you", said Mr. Koto "This is Bruce and this is Alice. They are from overseas. You will have heard, surely, that there was a plane crash. These children made it to our shores in a small dinghy! They have been our guests ever since. They are getting quite used to Ruritanian ways, and we thought we would bring them to EIGHTLAND and show them all the beautiful things you have in your land!"
"It's a pleasure to have you" said the superintendent "Let me invite you to lunch"
"Thank you very much, we had something to eat on the boat. I am sure the children are not hungry, but thanks all the same" replied Mr. Koto.
"In that case, just make yourselves at home. Perhaps you would like to see what the children are doing?", suggested the superintendent.

It was decided that they should visit some of the children's classes, and then look around in the Sports ground and in the Recreation room.
"What is a recreation room?" asked Alice.
"That's the place where the children play some games" replied Mr. Koto "they might not have learned all the rules well enough and they can practice in there. Some of these are card games, like the game with the red and green tickets. Here they have red and green tickets, some being large and others small, but here they have the added complication of some tickets having holes in them and some not. They play a game with these tickets that has the same rules as one of their tennis games"

They went into the classrooms, where some children were playing some of the mathematical card games, others were painting, and others were studying their books. Then Mr. Koto took the children to see the swimming pool and the tennis courts, which were indeed divided into eight triangles. They watched the tennis for a while, there were eight children playing and they seemed to be having a good time. They hardly ever dropped the ball, but when they did, they immediately knew who was to serve next. Bruce knew how hard it was to work out who was to serve next from knowing who served last and where the ball dropped. He could not work out the rules just by watching the children play. There were of course one-zero courts (eight, as we would say), so there were one-zero ways to serve the ball and one-zero ways of dropping it, which made one-zero-zero possibilities (or sixty-four, as we would say it)
"I hope we can have a game of tennis here", said Bruce "I'd really like to learn the rules of at least one of the games in this land!"
"You can play tennis, if you like" said Alice "I think I'll go for a swim. There is the swimming pool! Look! It's in the shape of an octopus!
"Oh yes!" said Bruce, "Isn't it fun? There are eight equal triangles painted on the bottom. I wonder if they use the pool for racing?"
"Yes they do" said Alo "Do you see that little round island in the middle of the pool? They jump in from there, all one-zero of them at once, and each person swims to the side of one of the eight triangles. I think I'll have a swim with you" said Alo to Alice.

So Alo and Alice went to the swimming pool and Mrs. Koto and the Koto girls went to the library, while Bruce went on watching the tennis. When the game was over, one of the children asked Bruce if he would like to play. He said he would stand out, giving him his place on one of the courts. The rules were really quite simple, they were explained to him like this:

The courts were marked with the numerals
0, 1, 2, 3, 4, 5, 6, 7
going round in the clockwise sense. If the ball fell in a court with a particular number, you had to count that just that number from the server's court, in a clockwise sense, to know who was to serve next. For instance if the ball was served from court 3 and the ball was dropped in court 2 , then the next server was the one in court 5. Or if the server was in court 5 and the ball dropped in court 4, then the next server would be the one in court 1, because counting 4 courts clockwise from court 5 you got to court 1.

Bruce picked this up very quickly and they played several games. He did not win any of them but he enjoyed playing them. Then he offered his place back to the child who had stood out for him.

He stayed a while watching, but he could not make head or tail of the service rules. He realized that they had switched to another type of tennis and he did not yet know these other rules. The children had called it the two by two by two game and Bruce was determined to find out how you played it.

In the meantime, Alo and Alice were having a wonderful time in the swimming pool. The water was beautifully warm and clear, it was water from the sea, they had channeled it in from the bay into the pool. There were many other children in the pool who were staying in the prison and soon they made friends with them. They talked a lot with a boy whose name was Okto, a popular name in EIGHTLAND, it must have something to do with the number eight!

How do you like it in prison?, asked Alice.
It's not too bad", replied Okto, "I'm getting a little tired of it now, but I don't really want to go home until I've learned everything. It would be silly to go back to school without having learned what I came here to learn"
"What do you have to learn?", asked Alice.
"Some of the eight-rules", replied Okto "I know quite well how to play some of the eight-games, like the water polo, tennis or card games that have these rules. There is one that I'm not very sure of. There are five, you see, and I'm fairly sure of four of them but I still need to learn the last one."
"Do you mean that you decide yourself when you go back to your normal school?", asked Alice incredulously.
"Yes, usually", said Okto "Of course the superintendent does have the last word, but if you insist you want to go back, he always lets you go. But usually people don't want to go home until they have learned what they came here to learn. Some come here for very selfish behavior, they seem to stay a bit
longer than those that come to learn the rules. It seems more difficult to learn how to behave unselfishly. As far as I am concerned, I'll probably stay another two or three more days and then I'll go home. And it's nice to have the pool and the library, it's really better than being at a normal school."
"Are there enough people around to play an EIGHTLAND water polo?", said Alo to Okto.
"I think I can rustle up an eightsome!", said Okto "Let me see, there are two girls over there and $I$ can see three boys just coming out of the building, I'll go and ask them"

They soon got an eightsome together and they played water polo, following the same rules as Bruce's first tennis game. The idea was to keep the ball in the air, so that it did not touch the water. If it touched the water in one of the "courts", then the number of that court and the number of the server's court would decide who would serve the next ball.

Alo and Alice soon caught on to these rules. It was good exercise and in about half an hour they finished three games. To win a game a person had to reach one-zero points. After playing these games, they decided to get dressed and to find the others.

By this time it was getting late in the afternoon and it was time to go somewhere else to have something to eat. The superintendent again invited them to stay for a meal; he was pleased to have some out of state visitors and he liked the idea of explaining to Bruce and Alice about how they ran their prisons. The Kotos accepted the invitation, so they all sat down with the prisoners and had a good meal together. They only had a little way to go to the place where they had booked in for the night, so the children were told that they could roam around for a little while. Bruce and Alice came upon a little girl who was holding up a square in the sun and casting shadows on a large piece of wood in all sorts of ways.
"That looks like a good game", said Alice to the little girl "What's it for?"
"I'm learning geometry", replied the little girl.
"Geometry?", said Bruce, "I thought geometry was about circles and angles and theorems!"
"Oh", said the little girl " I've never heard of theorems, but what I have to find out is what kinds of shapes the shadow of a square can cast when you hold it in the sunlight. You have to know how to hold the square and how to place the piece of wood for the shadow so that the shadow becomes a square, or a more general rectangle or a rhombus which is not a square, or a general parallelogram"
"Oh", said Alice nonplussed "I suppose that sounds like geometry, but are you sure you haven't learned any theorems?"
"Quite sure", said the little girl "What's a theorem?"
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"I'm not too sure myself:, said Bruce "but it's something you have to learn in geometry. They have proofs and things and when you've finished you have to say Q.E.D."
"We perhaps don't do geometry the way you do", suggested the little girl "Anyway, come and look at my square. Take it and hold it so that the sun falls directly on to it. I think it's called the rays being at right angles to the square. Then place the other large piece of wood so that it is parallel to the square. You'll find that the shadow is also a square. Now turn your square gently so that one side of the square comes nearer to the piece of wood, while trying to keep the opposite side where it is, then the shadow will be a rectangle which is not a square. Now if you place the two pieces parallel again, but hold the square by two opposite corners, then tilt one corner towards the large piece of wood, the shadow will be a rhombus, but no longer a square. I can easily make the shadow into a general parallelogram, which is neither a rectangle nor a square nor a rhombus. I have not been able to make opposite sides unequal, they always seem to stay equal. Opposite sides always seem to stay parallel too, So whatever I do, I always seem to get a parallelogram for my shadow."
"That's very interesting", said Bruce "Let me try and turn it around a bit longer. I think what you are finding out must be a theorem!"
"But you haven't told me yet what a theorem is!", objected the little girl.
"It must be something that is always true", replied Bruce a little uncertainly.
"Oh, I didn't know that there were any such things" replied the little girl "because what is true in one land is not true in another land. When you go across to NINELAND, everything is different, and whatever we've been taught here, we have to unlearn when we go there!"
"You may have something there", said Bruce "but there are certain things that are true here which would stay true even if you went to NINELAND"
"Would there really?", said the little girl in amazement "that's very nice. I've often wondered about that."
"I think what you've found out with your square you are playing with in the sunlight would still be true in NINELAND!", said Bruce emphatically.
"I suppose it would", came the reply "but I don't suppose you'd be allowed squares in NINELAND"
"I suppose you'd have to have triangles or nine pointed stars made with three triangles. Perhaps even triangles are not allowed, since they don't have nine sides, oh I beg your pardon: one-one sides"
"I'm not very sure", said the little girl "I've only been over once, and it's very muddling. I don't want to go again until I've learned a great deal more about it. We don't get much chance of learning about it here, since it is against the law here. In fact this is why I am in prison", the little girl was almost crying now, her lips twitched a bit, but she controlled herself and went
on "because when we came back from NINELAND, I got muddled up and counted in nines at school so they thought they'd better send me here so that $I$ could get into the swing of the eights again!"
"Poor you!", said Alice compassionately.
"Oh no!, I've quite enjoyed it" said the little girl "I think I'll be going home soon though. My mother is coming for me tomorrow, I think." At this point Alice had to leave the little girl with her square in the sunlight, since it was time to go. She said goodbye to her and wished her good luck. They also said goodbye to all the other children they had been playing with, as well as to the superintendent and then climbed into the car.

They drove a few EIGHTLAND sextas along the road, until they reached a plateau, where there was just one building lying half-hidden amongst some trees. The plateau seemed to be very rocky, as though somebody had poured some huge rocks all over the place. They noticed that jets of steam were coming out from between some of the rocks.
"What's that steam?" asked Bruce and Alice together.
"That's volcanic steam", replied Mr. Koto " there is a volcano just behind there, and sometimes it erupts"
"Goodness! Is it dangerous?", asked the children.
"No, it isn't", replied Mr. Koto "You don't get dangerous eruptions these days. There was a big one many years ago, but it's been quiet for a long time. You get occasional small lava fountains, but sulfuric fumes are always coming out between the cracks in the rocks. And there is always hot steam coming out " he said, pointing to a big jet of steam shooting up into the air from the ground "You can go and feel with your hands, but don't get too near, it might burn you."
"I don't think I want to try it", said Alice.
"I'll go", said Bruce, considering himself very brave!
"There is a smaller jet over there" suggested Mr. Koto, pointing in another direction, near one of the big rocks.

Bruce got out of the car and walked down to where the smaller jet of steam was coming out of the cracks between the rocks. "Yes it is hot!", said Bruce "You can get your hand in so far, but then it gets too hot and you can't bear the heat!"
"Let me try!", said Alice, emboldened by her brother's apparent bravery. So Alice tried as well and they tried to vie with each other as to which of them could reach down deeper into the steaming crack, but beyond a certain point they just could not stand the heat.

The hotel was not far from that spot, they drove through the entrance gate and up to the front door. It was a good looking hotel, an old fashioned, but friendly looking building. So they checked in for a couple of nights as Mrs. Koto wanted to show the children a lava flow and other interesting things to do with volcanoes. One or two of the craters were quite near the hotel, where some
of the recent eruptions had taken place. Most of these were inside one big crater. That big crater was the one left behind by the last really big eruption many, many years ago. So this volcano was really safe, as the big crater sheltered people from the eruptions that happened inside it. Occasionally small lava fountains broke through in the surrounding jungle, but this happened but rarely, and the rangers were always there to warn you if there was any danger.

In the hotel Bruce and Alo had one room, Ata and Unta and Alice shared another and Mr. and Mrs. Koto had the third room. As Alice was getting her things out of her night case, she noticed that the picture on the wall was moving.
"Look at that picture", said Alice, "Why is it moving?"
"Oh, that's all right", said Ata "it's only a little earthquake."
"An earthquake?" said Alice "Do you just talk about it as though nothing was happening?"
"Yes of course", said Ata "They never hurt."
Alice was learning all the time. Then Bruce was knocking on the door.
"What's the matter?", asked the girls.
"What's happening? What's all that rumbling?", asked Bruce in rather a concerned voice.
"It's only an earthquake", shouted the girls through the door "It doesn't hurt"
"Oh", said Bruce flatly and retired in good order.
By the time their evening cup of chocolate was brought in, the earthquake had stopped and all was quiet. The children discussed their day. Bruce and Alice realized that they were learning a lot of things. They had learned a great deal of mathematics by getting used to the rules of the different States. They had learned about the children's prisons and wondered about these different ideas about how to run them. They had also learned about volcanoes, even about small earthquakes, and steam coming out of the ground from between the rocks. What a lot they would have to tell their friends when they were back in their own country!

They spent several more days in EIGHTLAND, exploring around the big crater, the jets of steam, they even had a few short walks on the edge of the jungle, but they were not too keen to go into it too deeply! They visited the prison again before going home. The little girl with the square had already gone home but there were enough children around so that they could play tennis and water polo. Bruce was able to learn one more tennis game, the one that was similar to the game with the red and green tickets. The other three games seemed a bit too complicated, but Okto had written down the rules for him, so that he could look at them at leisure when he got back to THREELAND. He would have to learn them on the quiet, as they would be illegal in THREELAND. Or of course he could wait till he got back home.

When they got home to THREELAND, there was a telegram waiting for them from their parents, which had been brought over from Fourville by express mail. They would be in Ruritania in another two days and would take them home!

Imagine their welcome when they reached home in their own country! They spent a lot of time explaining to their friends how things were done in Ruritania. The teachers became interested in the unusual ways in which Ruritanian children learned their mathematics. So there was hope yet for school to become more interesting, and Bruce and Alice would help the teachers make up interesting mathematical games, which would amuse all their friends as well as teach them a great deal.

## MORE THINGS TO DO AND TO THINK ABOUT.

(1) Bruce and Alice never managed to get to FIVELAND. But you can pretend that you are in FIVELAND and work out the following:
(i) How would they measure time? Would they divide the day and the night into twenty-five equal parts? If so, would their "hour" be longer or shorter than hours? What would be their "minutes? Would twenty-five of their minutes make one of their "hours " Or maybe fifty? Would they be longer or shorter than our minutes?
(ii) Make up the rules for a FIVELAND tennis or water polo game. The court would have to be pentagonal, wouldn't it? Could there be more than one type of tennis?
(iii) Work out how they measured distances in FIVELAND. If a una is still 3 cm long, a dua would have to be 15 cm . Work out their tria, their quarta, their quinta and so on. Which would be nearest to a kilometer?
(iv) Draw a map of Fiveville. It will have to have five main streets, which would be called Zero Street, One Street, Two Street, Three Street and Four Street. What shape would the central figure be which formed the roundabout in the middle of the city?
(v) They had a card game with twenty-five cards in it. On every card on one side there were some flowers, red ones and/or blue ones, and on the other side there were numerals. There were never more than four of either color flower on any card, and all the numbers used were less than one-zero-zero (understood in Fivelandish!). They used the zero.

The rules for putting the numbers were these: Count the red flowers. The number you get will be the first digit of the numeral you are looking for. Count all the flowers, multiply this number by 2 . The last digit of this Fivelandish number will be the last digit of the numeral you are looking for.

Make a set of cards like that and try to see whether you can tell how many of each color flowers there are on the card, by just looking at the numeral on it.
(2) Here are the rules for the first Eightland Tennis Game:

| The Itall |  | 日 | 1 | 2 | 3 | 4 |  | 6 | 7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| cerved by | 0 | $\square$ | 1 | 2 | 3 | 4 | 5 | 5 | 7 |
| served by | 1 | 1 | 2 | 3 | 4 | 5 | $E$ | 7 | 0 |
| serred by | 2 | $z$ | J | 4 | 5 | 6 | 7 | $\square$ | 1 |
| carvad by | 3 | - | 4 | 5 | 6 | ' | 4 | 1 | 2 |
| servel liy | 4 | 4 | 3 | 6 | 7 | 0 | 1 | 2 | 3 |
| served by | 5 | 5 | 6 | 7 | 0 | 1 | 2 | 3 | 4 |
| served by | 6 | 5 | 7 | 0 | 1 | 2 | $\exists$ | 4 | 5 |
| served by | 7 | 7 | 6 | 1 | 2 | 3 | 4 | 5 | G |

The numbers in the squares tell who is to serve next.
Here are the rules for the second game. Small letters mean small tickets, big letters big tickets. The star * means a hole.

|  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| serwed by | 09 | $\mathrm{g}^{\text {可 }}$ | $G^{1}$ | $\mathrm{r}^{2}$ | R | $y^{4 *}$ | $\mathrm{G}^{-3}$ | $\mathrm{I}^{-4+}$ | $\mathrm{R}^{7} \mathrm{H}$ |
| cervad by | 16 | ${ }_{6}^{1}$ | $\mathrm{f}^{41}$ | $\mathbf{R}^{3}$ | $\mathrm{r}^{2}$ | $G^{5}$ * | $4^{4 *}$ | $\mathrm{R}^{\text {\% }}$ | ${ }^{4}$ |
| serwed by | 21 | $\mathrm{r}^{2}$ | $\mathrm{R}^{3}$ | $9^{0}$ | $G^{1}$ | $\mathrm{r}^{6} \mathrm{x}$ | $\mathrm{R}^{7 \times}$ | $\mathrm{g}^{4}$ | ${ }_{G}{ }^{4}$ |
| served by | 3 R | $\mathrm{R}^{3}$ | $\mathrm{S}^{2}$ | $G^{1}$ | $\mathrm{g}^{10}$ | $\mathrm{R}^{7}$ * | $r^{6}{ }^{*}$ | $6_{5}{ }^{5}$ | ${ }^{4}$ |
| serwed by | $\begin{aligned} & 4 g^{H} \\ & 5 G^{\#} \end{aligned}$ | $9^{9^{\prime}}$ | $\left\lvert\, \begin{aligned} & 4^{5} y_{y} \\ & y^{4} \end{aligned}\right.$ | $\begin{array}{r} \mathbf{r}^{\mathbf{E}_{\mathrm{m}}} \\ \mathbf{R}^{7} \end{array}$ |  | $\begin{aligned} & 9^{0} \\ & G^{1} \end{aligned}$ | $\mathrm{y}^{1}$ | $\mathrm{r}^{2}$ $\mathrm{R}^{3}$ | $\underbrace{n^{3}}$ |
| cerved by | $6 \mathrm{r}^{\text {* }}$ | $r^{\text {b }}{ }^{\text {a }}$ | $\mathrm{R}^{7}{ }^{\text {\% }}$ | $g^{4}{ }^{-}$ | $G^{3}{ }^{*}$ | $\mathrm{r}^{2}$ | $\mathrm{H}^{-1}$ | $5^{4}$ | $G^{1}$ |
| serwed by | $7 \mathrm{R}^{+}$ | $\mathrm{R}^{7 \mathrm{~m}}$ | $\mathrm{C}_{4}$ | $G^{5} \mathrm{H}$ | ${ }^{4}{ }^{\text {K }}$ | $\mathrm{R}^{3}$ | $r^{2}$ | ${ }_{6} 1$ | 0 |

The numbers in the squares tell who is to serve next.
The letters in the squares refer to the red and green tickets, large and small, with or without holes.

Find a way in which it would be easy to remember the above table.
The Eightlanders had an 8-clock, so their hour was like three of our hours. Their minute was one sixty fourth of their hour. How many of our minutes would that be? Can you use one of these tables for telling what the time will be after a certain number of hours in Eightlandish time keeping?
(3) Get 36 small cubes and make a box.
(i) Express the length, width and height of your box, using Eightlandish numbers.
(ii) How many "floors" are there in your box? How many cubes are there on each floor? Would it be 17 or maybe 12 or would it be only 6?

Do the same by taking 52 cubes.
(4) Here is a table for one of the remaining tennis games. Try to play it on a cardboard court.

| served by | 0 | 6 | 1 | 2 | 3 | 4 | 5 | 5 | 7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| served by | 1 | 1 | $\square$ | 3 | 2 | 5 | 4 | 7 | 6 |
| served by | 2 | 2 | 7 | 4 | 1 | 0 | 3 | 0 | 5 |
| served by | 3 | 3 | 6 | 5 | 0 | 7 | 2 | 1 | 4 |
| gerver hy | 4 | 4 | 5 | $f$ | 7 | (a) | 1 | $?$ | 3 |
| served by | 5 | 5 | 4 | 7 | 6 | 1 | $\square$ | 3 | 2 |
| served by | 6 | 5 | 3 | 0 | 5 | 2 | 7 | 4 | 1 |
| served by | 7 | 7 | 2 | 1 | 4 | $\square$ | 5 | 5 | $\theta$ |

They would have to have a hexagonal court, divided into six triangles, marked $0,1,2,3,4,5$. You can probably invent the one that is based on their thirty-six clock, since the Sixlanders divide their day and night into thirty-six or

$$
100 \quad \text { equal parts, }
$$

which they call one hour.
The other game would be something like the third EIGHTLAND tennis game, but of course with only six courts.
(6) Here are eight houses:


The name of each house is written below it.

```
    Arrange seven of the houses in a circle, putting the Z house in the
middle. Try to arrange them in such a way that there are three chimney houses in
a series of three next to each other, and that there are three door houses in a
series of three right next to each other, and also three two-window houses in a
series of three right next door to each
other.
```

Could you play a game with the houses that would be similar to one of the tennis games that they played in EIGHTLAND?

