## Chapter III

## Shopping in FOURLAND.

The next morning when Bruce woke up, he was surprised to find Alice crying.
"What's the matter, Alice?", asked Bruce in a concerned voice, "Are you worried about going to school?"
"NO!", sobbed Alice, "but I want to go home!"
"Go home? You are not serious?", said Bruce "just as it is beginning to be fun!"
"Don't you care about what is happening at home? Mum and Dad must be worried to death about us, they must think we are dead by now!", cried Alice, sobbing her heart out.
"Do stop crying, Alice, it won't do much good!", said Bruce in a somewhat embarrassed voice. He heaved a big sigh, got out of bed and went to sit next to Alice.
"I'm sorry, Alice", he continued, putting his arms around her affectionately "I spoke without thinking! We've had so many adventures, which l've found great fun, and what with one thing and another, I clean forgot about our parents! I have an idea though, let's send them a long telegram, and tell them all that has happened and that we are all right."

There was a knock on the door, it was Unta.
"It's a beautiful day!" said Unta, "and since today there is no school, we thought we'd take a trip. Would you like to come too?"
"We would, very much. " said Bruce.
"Would you like to have a little tour of FOURLAND?", said Unta.
"That would be wonderful", replied Bruce, "but before we go, could we send a telegram to our parents?"
"That's exactly why we thought of going to FOURLAND", said Unta, "My mother said that your parents must be very worried, and it is only from FOURLAND that you can send telegrams abroad."
"I'm so glad", cried Alice, wiping her tears, "we were just talking about our parents, and wondered what they must be thinking. "
"What's it like in FOURLAND?", asked Bruce "apart from the foreign postal service. "
"It's bigger than THREELAND", said Unta, "and there is a big city called Fourville where there are a lot of stores and a big supermarket. You can buy almost anything there! My mother thought that you would like to do a bit of shopping and send your telegram while we are in the city. "
"When are we going?" asked Alice, hopping from one foot to the other in her excitement.
"We can go as soon as you are ready", replied Unta.
"But we have no money!", interjected Bruce.
"Oh yes we do", said Alice, "Dad gave us some US Dollars and some travellers cheques, I have saved them in this inner pocket", and so saying was pointed to a secret pocket within her dress.
"Don't worry!", said Unta, "Mum said she could easily lend you some local money, and you can pay us back when your parents come to collect you!"
They left soon after breakfast. The road wound through the mountains towards the FOURLAND border. Alo explained that the border followed a range of mountains, with TWOLAND and THREELAND on one side of the mountains and FOURLAND lay behind the range. The other States were on smaller islands around the Main Island.

The road took them higher and higher, through dense forests and rocky gorges, taking an interminable series of hairpin bends. Eventually they left the forest behind, and all they could see was pastures, dotted with rocky outcrops, although there were flowers of all sorts of colors everywhere. Soon they reached the pass, the highest point on the road they were taking. The pass was the border between the two States.

There was another building, lying a little away from the road. There was a notice on the door about the exchange rates, and it was a bit different from the other list. The red, the orange and the yellow were the same, but the blue and the green were different. This is what Alice noticed:
Fourland Threeland
currency
1 blue
1 green
currency

1 blue, 10 green, 12 red, 10 orange, 1 yellow
1 green, 3 red, 3 orange, 1 yellow

She realized that 10 must mean four and that 12 must mean six, since they were in FOURLAND.
"I believe the two notices really say the same thing", suggested Bruce. He took his notebook out and wrote the THREELAND currency equivalent to 1 blue on one of the pages like this:

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b gggg rrrrrr oooo y
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saying: "and making the exchanges in Threelandish fashion, we have:"
and he wrote:

|  | b | g g g g | rrrrrr | 0 |
| :---: | :---: | :---: | :---: | :---: |
| and then | b | g g g g g g | $r$ | 0 |
| and then |  |  | $r$ | 0 |
| and then |  |  | $r$ | 0 |

"which is what we saw on the other notice!", finished Bruce.
"I'll do the same for the green coin!", said Alice excitedly. She did so, and the two "amounts" were in fact the same!
"Now we know how to turn THREELAND money into FOURLAND money", said Alice, "but we don't know how to turn travellers cheques into Ruritanian money!"

Mrs. Koto overheard some of this conversation between Bruce and Alice so she said to them:
"Don't worry about the money! We can settle all that later when your parents arrive, which I am sure they will do quite soon"

At first they had some problems since Bruce and Alice had lost their passports in the accident, but the guards had heard of the accident and so let them pass without any trouble. They had to rearrange the seating at the border, because of the different regulations in force in FOURLAND. Bruce, Alice, Alo and Unta went to sit together on one of the bench seats, Mr. and Mrs. Koto sat in the front, but not too close to each other, and Ata, poor thing, sat in the back with the luggage! In FOURLAND you were not allowed to have three people sitting together!
As they descended on the FOURLAND side of the mountain range, they noticed that the houses were grouped in fours, and the clumps of trees near the houses were also in fours.

The fields seemed nearly all square shaped, whereas in THREELAND most of the fields were triangular in shape. In the four corners there were usually four trees. In some orchards there were sixteen trees in each corner with a little space left in the middle for collecting the fruit.

Alice was happy to realize that she could switch quite quickly to thinking in fours, instead of thinking in threes. This was made easy by the look of the countryside, which all seemed to be organized around the number four!
"Are we getting to the stores soon?", asked Alice.
"Yes, we'll soon be there", said Ata, "the biggest one is in the center of the city, and it is on several floors. But we'll go to the Post Office first, so that you can send your telegram. "

Bruce had been daydreaming a bit and he started dreaming about going home. He wondered how long they would stay in Ruritania. He put the question to Mr. Koto.
"I don't really know how long you'll be staying! You are certainly welcome to stay with us as long as you like", replied Mr. Koto "You see this island is very mountainous and we have not yet been able to build an airstrip for jet planes. Most people still travel by sea, unless they have a private aeroplane and possibly a private airstrip! As far as I know, the next boat is due in about two or three weeks' time. "

Bruce did not say anything, but he thought a couple of weeks or so in Ruritania would suit him very well. He was really in no hurry to leave, he thought it was fun to be in Ruritania; Alice did not share Bruce's views on this. She was more homesick than Bruce was and she was less enthusiastic about the rules to do with the numbers which they had to obey, or else! But she was beginning to see how the different number systems worked, as Bruce had explained to her, it was not all that different from what they were used to, they just had to replace the number ten by some other number.

Bruce began to wonder how they would handle the money and how they would measure distances. He wondered whether they would have kilometers or some other ways of measuring distances between cities.
Mrs. Koto noticed that Bruce had become a little thoughtful and so asked him if there was anything the matter.
"No, it's nothing," said Bruce, "is it legal to use THREELAND money here in FOURLAND?"
"No", replied Mrs. Koto, "But we have some FOURLAND money we can lend you. We changed some at the border. "
Mr. Koto was slowing down. Bruce looked out of the window, and realized that they were coming to a stop just outside a Post Office. When the car had stopped Mrs. Koto said to Bruce:
"Here is some FOURLAND money, go and send your telegram. I am sure you can do it by yourself!"
Mrs. Koto handed Bruce some coins and motioned to him to get out of the car so he could send his telegram. The coins seemed like ferry tokens, and they were in all sorts of colors. He tried to remember the notice he had seen at the border.
"I wonder what all that is worth?", said Bruce to Alice, who was also getting out of the car to go with Bruce to the Post Office.
"I haven't the least idea!" said Alice, "but you can bet your bottom dollar that each coin is worth four times as much as one of the other coins of a different color!"
"You are getting really smart", said Bruce, admiringly, "But what I need to know is how much each of them is worth!"
"There is one way to find out" said Alice "Let's go in the Post Office and see!"
They took a telegram form from one of the shelves, wrote out briefly what had happened to them, ending the telegram with the words "Don't worry, we are having a good time"
The man behind the counter counted the words and said:
"One two three zero"
Bruce blushed, he hesitated a moment, and then put the whole heap of coins on the counter, suggesting that the man behind the counter help himself.
"You must be foreigners", the man said with a smile "That will be one green, two red and three orange, all right?"
"If you say so", said Bruce, "Here you are" and so saying he gave the man behind the counter one green token, two red tokens and three orange tokens and put the rest in his pocket.
"That's correct", said the man.
"Excuse me please", said Bruce, "could you tell me which color has the least value?"
"It's the yellow", replied the man, "It's the yellow coin which is worth the least in all the States. Perhaps you are not from these parts?"
"We only arrived a few days ago", put in Alice, "and we are trying our best to get used to it!"
"I'm sure you'll do fine!", said the man.
As soon as they returned to the car, Mr. Koto said he would take them to the central store, the biggest in the City and would leave them there to get on by themselves. Mrs. Koto said to Alice that she should buy one or two dresses, as she had nothing to wear except what she had on. Bruce wanted to buy some water colors, which he would share with Alice, as they both liked painting. They both thought it would be good to paint some of the things they had seen, so they would have some souvenirs of Ruritania.
"I am sure you will get on very well by yourselves", said Mr. Koto, "you seem to be reasonable children and I am sure your parents would wish you to get what you need. We can lend you any money you need. "
Alice clutched the coins firmly that were given to her, she certainly did not want to lose them! Bruce took her by the hand and they bravely entered the big store, taking the elevator to the third floor, where they were selling dresses for young girls.

Alice looked for a long time amongst all the lovely dresses that were hanging in the store and finally chose one blue and white dress for everyday wear and a pale green one for special occasions. She was happy to have something to change into, as the only dress that she had was much too hot for the tropical climate of Ruritania. She took the dresses to one of the cashiers and asked how much she had to pay.
"That will be one two mauve and three blue, if you please", said the girl at the cash desk. Alice was not sure what the cashier meant at first, but then she remembered that "one two" here meant six, so she produced six mauve coins and three blue ones. This was duly accepted and the dresses were wrapped up and given to Alice.

She soon realized that after this purchase she had no more mauve coins, each of which must have been worth quite a bit. Six mauve coins and three blue coins were enough to buy her dresses! She found it hard to realize that these little tokens were real money! She looked at the coins she had left; she still had some blue ones, quite a lot of green ones, some red, some orange and some yellow. She knew from the post office that the yellow ones were worth the least, and now she realized that the mauve ones must be worth the most, very likely followed by the blue ones next down in value.
"Do you know the value of the coins yet?" asked Bruce.
"Yes, I think so", replied Alice, "the mauve ones must be the most valuable, since I needed those to buy the dresses. After the mauve come the blue, I think, and then the green, the red, the orange and the yellow, in that order. Do you think I have them right?"
"You are really getting good at this" said Bruce, "you mean that four yellow ones are worth an orange, four orange ones are worth a red, four red ones are worth a green, four green ones are worth a blue and four blue ones are worth a mauve!"

Now they had to find the Department where they could buy their paper and paints. They realized that they had forgotten to ask Mr. Koto about the units of measure, so they would have some problems to ask for the size they wanted. There was nothing for it but to try! They went up to a counter and asked a salesman to show them some paper. They were shown some paper, but they both thought it was too small for what they wanted.
"What size would you like?", asked the salesman.
Bruce was dumbfounded for the moment. Alice, however, noticed Ata and Unta at a nearby counter, who were discussing something they were wanting to buy.
"Look, Ata, we don't know your measures. Could you tell us what we should ask for?", said Alice, showing with her hands the approximate size of paper they wanted.
"Maybe you want pieces two trias by three trias, or maybe a square shape would be better, in that case ask for pieces two trias by two trias", suggested Ata.
"I have no idea what such a size would be", objected Bruce.
"I'll explain the measures to you later", said Ata "just ask for it, I'm sure it will be all right"
The children did so and were shown a sheet which they guessed would be about 1. 20 meters by 1 meter. Bruce thought that might be too big, so he said:
"I'll have some sheets two trias by two trias"
"How many sheets would you like?", asked the salesperson.
Bruce made a rapid calculation and then said:
"One one zero please"
Bruce had worked out that 100 was sixteen, and a further 10 was four, so if he wanted twenty sheets, he would need to ask for 110 sheets.

The salesperson counted out twenty sheets, which only came to three red. Bruce paid with three red tokens, then they both went over to Ata to ask about the measures.
"In FOURLAND they have the measures una, dua and tria. I believe one una is about one inch, at least that is what our teacher told us the other day. There are four unas in a dua and four duas in a tria"
"One English teacher told us that in England and in America they use inches. I believe they use something they call a foot, which is as long as twelve of their inches. Then three feet are equal to one yard. So far so good, but after this it really gets involved! Two hundred and twenty yards (is that how you say two two zero one one in your country?) make one furlong, and eight furlongs are equal to a mile. It seems to me very hard, how do you manage to remember all these different numbers?"
"On the Continent of Europe and in Canada we use a system that is something like what the law prescribes here in TENLAND" replied Bruce. "Ten centimeters are equal to one decimeter, ten decimeters make one meter, ten meters are equal to a dekameter, ten dekameters are equal to one hectometer, and ten hectometers make one kilometer. The money also goes in tens. In England the money used to go in twelves and twenties, but now they have changed to tens, as they have in most other countries. "
"It is a lot easier in Ruritania", admitted Alice, "in each State you only use one and the same number!"

Bruce was trying to compare the FOURLAND units of measure with those used in England and the United States as well as with those on the Continent of Europe and in Canada. The sheets he had bought measured about a yard square, a yard being just under a meter. That meant that a tria would be about 45 centimeters. He never got any further with his calculations, since he suddenly realized that he had not yet bought the water colors. So he went back to the same counter and the same sales person greeted him.

I suppose you would like some paints", said the sales person, "do you use water colors or do you paint in oil?"
"I'd like some water colors in tubes, please", said Bruce.
The sales person produced a large assortment of tubes and put them all on the counter. Alice, who was standing by him cried out: "Oh, aren't they lovely! I would like some deep red colors, some orange, but also some shades of green and blue!"
"We'd better see how much they are", said Bruce practically, "How much are these tubes?" asked Bruce.
"These are two red and three orange each; here is a complete set, you pay less per tube that way. The whole set is only one green. The brushes, average quality, are selling at one red two yellow. "
Both the children were trying hard to work out what all that would be in dollars or in Pounds or in French francs, but they did not know the rate of exchange. They would have to simply keep their purchases to the handful of coins that they still had left! Bruce guessed that a yellow might be worth 50 cents, Alice disagreed, she thought it was more likely to be worth about ten cents or even less!
"We'll take the set for one green and we'll have one two brushes at one red two yellow each. That will come altogether to . . . . "
"Two green two red and three orange", said the sales person, having worked it all out in his head, without touching his hand calculator.
Bruce was amazed. The sales person never wrote anything down, he had done it all mentally. Bruce tried to be as good as the sales person, but Alice was working on it too. She was trying to do it in the following way:

First of all, four brushes at one red two yellow, would be one green two orange. Another two brushes would cost two red and four yellow, which is two red and one orange. So all the brushes together would cost one green, two red and one orange. Adding the one green for the paints, the whole cost is two green, two red and three orange.

[^0]"Look here, young man", the sales person said somewhat crossly, "I have just told you the amount, and I am not in the habit of making mistakes!"
"It isn't that" said Bruce, somewhat confused, "but I am not used to your way of counting!"
"Oh, I see, you are from another State", replied the sales person, "You should have said so before, and I would have helped you work it out. It seems that your little girl friend here has already worked it out!"
"She is not my girl friend!" retorted Bruce, "she is my twin sister!"
"I see", said the sales person "I might have known. You are probably too young to have a girl friend! I am sure you can work all these things out very well between you, just take your time, you will both do fine!"
Mr. and Mrs. Koto were waiting for them in the car, Ata and Unta had also finished their shopping, and Alo was absorbed among the luggage in one of his favorite books. Mr. Koto suggested that since everyone had finished what they wanted to do, it was time to start back and they could look for a place on the way back where they could have their lunch. So they all made themselves comfortable, according to FOURLAND rules, and they started back towards the mountains.

They arrived at a pretty little village, where Alice counted one zero zero houses, and not unexpectedly, around each house there were one zero trees. The children soon worked out that there must be one zero zero zero trees in the village!
The whole village was set out in the form of squares. This gave the place a pleasant and orderly look. In the middle of the village there was a village green, covered with what looked like freshly made lawn. In the middle of this green area there was a square building with one zero shops. These shops sold souvenirs for tourists, sandwiches and cookies for those who needed refreshments.

They stopped in the middle of this pleasant village, sat down, using some of the benches provided for tired travellers! Mrs. Koto opened the cool box in which she had stowed some nice eatables for their picnic. Alice said she was thirsty and asked if she could go and get a drink.
"Certainly, my dear", said Mrs. Koto, "Do you have any money left?"
"Yes, thank you, Mrs. Koto", said Alice politely, "and I hope I shall know how to check the change!"
"Shall I come with you?", offered Bruce.
"No thank you, I want to see how I can do it all by myself" replied Alice.
Alice was at the store where they sold drinks in one moment, and bought a bottle of papaya juice. She looked at a little windmill, wondering if she could have it, but she was not sure if she ought to!
"How much is the fruit juice?", asked Alice.
"Two orange and one yellow", replied the old lady, whose store was full of all sorts of bric a brac.

Alice put a red coin on the counter and awaited developments.
"Good", said the old lady "I said two orange and one yellow, right? Three yellow make one more orange, and one more orange makes a red", and she placed on the counter one orange coin and three yellow coins.
Alice was a little taken aback at the rapidity of the old lady's calculations, and she was too embarrassed to check it, as she thought it would probably be correct! She thanked the lady and ran back with her papaya juice to where the Koto's and Bruce were already enjoying their picnic. She gave the juice to Mrs. Koto and put her change on a paper plate. Mrs. Koto poured out the drinks in small paper cups and they all enjoyed the fresh taste of the fruit. Then she asked Bruce if she had had the correct change. "Well, I suppose I can work it out like this", said Bruce, not really wanting to be disturbed in his picnic, "If the drink cost two orange and one yellow, three more yellow coins would make one zero yellow, which is one orange. Together with the two orange that makes three orange, so you needed another orange to make one zero orange, namely one red. So you must have had one orange and three yellow in your change, if you gave the lady a red coin"
"Yes", said Alice "that's what she gave me. It doesn't seem too hard to count in Fourlandish, does it?"
"Very good, Alice", said Bruce "but don't get too used to it, soon you will have to count in Threelandish again!"
"Yes, you are right!", said Alice "but I think I shall be able to switch quite quickly, and think of one zero as three, instead of one zero being four! Then I won't have to go to prison and have lessons! Don't you think we ought to ask Mrs. Koto if we could have a tour of Ruritania and visit TENLAND? At least there people would be counting in the same way as we do at home!"

Bruce did not reply as he saw Mrs. Koto starting to pack up the picnic things, so he helped her tidy up their spot, so they would leave it as clean and orderly as they found it.

After taking the bottle back to the old lady they all got in the car and started off towards the border. Bruce and Alice were very quiet, as they were both thinking over the events of the day. They were happy that they had been able to send a telegram. They could just picture their parents' faces when they learned that they were both safe and sound. Soon the boat would arrive in THREELAND and it would be time to say goodbye.
Bruce kept thinking that as far as he was concerned, there was no great hurry to leave! The Ruritanians were kind people, in spite of their weird habits to do with their own particular numbers! It would be a pity to leave before learning more of their habits. Alice was satisfied with her day too, happy that she had been able to do some shopping on her own, since the number-rules did not seem as strange as they did in the beginning. Everyone had been so kind and helpful, and she was beginning to feel at home.
When they had had supper and Bruce and Alice were alone together again, Alice said to Bruce:
"You know, Bruce, I think I've worked out an interesting pattern about how to get a red coin with the money of a country whose number is one less!"
"Oh yes? What is it?", asked Bruce, getting curious.
"If you take 1 red, 2 orange and 1 yellow TWOLAND coins, you can get one red THREELAND coin. But also with 1 red, 2 orange and 1 yellow THREELAND coins, you can get a red FOURLAND coin. And it goes on like that all the time!"

## SOME THINGS TO DO AND TO THINK ABOUT.

(1) Try to imagine a journey in FIVELAND, which everyone knows lies on a separate island. What sorts of units of measure would they have? What about their money? What would be the exchange rates, assuming the yellow was worth the same in all States?

Do the same for TWOLAND, or for SIXLAND or any other State that you think forms part of Ruritania.

The yellow coin is worth the same in all States. It is probably worth about 10 cents, or 5 pence, or about 40 French centimes. You will have noticed that the order of value of the coins is
yellow, orange, red, green, blue, mauve, ...
For larger purchases they also used black coins, which came after the mauve ones. Then there were larger coins as well. For example in FOURLAND, one large yellow was worth four small black, one large orange was worth four large yellow and so on.
People on the whole did not walk around with coins that were so valuable. Why do you think? How many small yellow coins would you need to change for one large black coin in TWOLAND? In THREELAND? In FOURLAND?
(2) In the whole of Ruritania, the smallest measure of distance was one una, which measured the same length in all the States. If you needed to measure very small objects, you needed smaller units of measure. One of these was the yellow unit. In THREELAND 3 yellow units were equal to 1 una, but in TWOLAND only 2 yellow units made a una. So the bigger the number of the State, the smaller the yellow unit was in that State. Then the orange unit was even smaller, the red unit smaller still, and so on.
The other units of length are easy to remember. Apart from the una, the length of the other units depended on the State in which you were! In FOURLAND 4 unas were equal to a dua, but in THREELAND, 3 unas already made a dua. In FOURLAND 4 duas were equal to a tria, 4 trias were equal to a quarta, four quartas were equal to a quinta, after which came the sexta, the septima, the ottava, the nona and the decima!

The distances between towns are measured in nonas or in decimas. Remember that when Bruce and Alice bought some paper for painting, they thought that two trias were about a meter long, so we could assume that a tria was perhaps 48 centimeters long. So a FOURLAND dua would be 12 centimeters and a FOURLAND una just 3 centimeters, which means that all the States start with a unit 3 centimeters long.

Try to work out how many meters there would be in a THREELAND nona, and then in a FOURLAND nona.

You could make a table like this:

| Name of measure | una | dua | tria | quarta |
| :--- | :---: | :---: | :---: | :---: |
| Number of unas |  |  |  |  |
| written in Threeland | 1 | 10 | 100 | 1000 |
| Number of centimeters | 3 | 9 | 27 | 81 |
| Number of unas |  |  |  |  |
| Written in Fourland | 1 | 10 | 100 | 1000 |
| Number of centimeters | 3 | 12 | 48 | 96 |

and carry on to quintas, sextas, septimas and so on.
Extend the table to include TWOLAND, FIVELAND and SIXLAND. Look at your tables for TWOLAND and FOURLAND; is there anything similar about them?
(3) In FOURLAND they measure distances between towns in nonas. Would it be just as convenient to measure distances in nonas in THREELAND? Don't forget that although a una has the same length everywhere, the duas are already different in the different States, a Threelandish dua is as long as 3 unas, but a Fourlandish dua is as long as 4 unas! So a Threelandish nona would be much shorter than a Fourlandish nona! Perhaps the Threelanders used the decima, or would that be too short too?

In most countries people use Tenlandish measures, since they use centimeters, decimeters, meters, dekameters, hectometers and kilometers, each one being as long as ten of the ones written before it.
(4) Let us suppose that your house or apartment was in TWOLAND. Which measures would you use for measuring the size of each room? Which units would you use for measuring the size of tables and chairs?

To do this in practice, cut out some strips of cardboard, each strip being 1 una long (3 cm ). Then make some duas ( 6 cm ), some trias ( 12 cm ), some quartas ( 24 cm ), some quintas ( 48 cm ) and some sextas ( 96 cm ). You could also make some yellow units (1.5 $\mathrm{cm})$. Then use these strips for doing your measuring.

Then make some Threelandish yellow units (1 cm), then some Threelandish duas (9 cm ), trias ( 27 cm ), quartas ( 81 cm ), quintas ( 243 cm ), and maybe even one or two sextas ( 729 cm ).
You could also make some Fourlandish and Fivelandish measures, if you have any cardboard left!
(5) Let us do some "shopping". In THREELAND you buy something that costs 2 red 2 orange 1 yellow
You put a green on the counter. What change do you get?
Next, you buy something that costs
1 green 1 red no orange 2 yellow
and you put two green coins on the counter to pay. What change do you get?
(6) This is a THREELAND bill
mauve blue green red orange yellow

|  | 1 | 2 | 2 | 1 |
| :---: | :---: | :---: | :---: | :---: |
|  | 1 | 1 | 0 | 2 |
|  | 1 | 0 | 0 | 2 |
| 1 | 0 | 2 | 0 | 1 |

Total to pay =

Try to work out the total.
(7) What would have happened if you had made the same purchases in FOURLAND, and by chance each article had been marked with the same numerals? These same numerals do not symbolize the same numbers! So if the bill looks exactly the same in THREELAND as it does in FOURLAND, then the FOURLAND bill is higher.

Work out the total imagining that
(i) you are in THREELAND, (ii) you are in FOURLAND,
(iii) you are in FIVELAND, (iv) you are in SIXLAND.

Do you get a different answer in each case?
(8) Imagine you are in Fourville and you want to buy some material. The material is sold in rolls, whose width is

1 tria and 2 unas .
You are supposed to bring home a length of
3 trias and 2 duas.
What area, in square unas, are you bringing home?
(9) If you have not been able to do the last problem, here are some hints:

Use some millimeter paper, and think of 1 millimeter as representing 1 una, so 1 square millimeter will represent 1 square una. One tria must be sixteen unas long, two unas added to this will give a width of eighteen unas. So you draw a length of 18 millimeters on your millimeter paper. Three trias must be forty eight unas long, another two duas
will add another eight unas, so the length will be fifty six unas. Now draw a rectangle 56 millimeters long on your base of 18 millimeters. The number of square millimeters in your rectangle will be the number of square unas in the material you have bought.
(10) Now, how much did all this cost in Fourlandish money?

Suppose that a square una costs one orange and one yellow. So you have as many orange coins as the number of square millimeters in your rectangle, as well as the same number of yellow coins. All you have to do now is to exchange these small coins for other coins of higher value.
(11) If you were actually living in FOURLAND, you would probably work out
$102 \times 320$ to get the number of square unas
of material you have bought, and then multiply this by $1 \quad 1$ to get the cost.
A Fourlandish child might do it this way:
$100 \times 320=32000$
$2 \times 320=1300$
so $102 \times 320=33300$
Then $33300 \times 11=$
$33300 \times 10+33300 \times 1=$
$333000+33300$ =
1032300
so you see the material was quite expensive, since you spent
1 black, 3 blue, 2 green and 3 red coins!
(12) In FIVELAND you want to share 420 books among five people (that is 1 0 people).

How many books does each person get?
(13) Here is another TWOLAND game. You play it with these one zero zero zero zero cards (sixteen cards, for you and me!)


You will need a piece of cardboard divided into four rows and four columns, and a small object to move around on it. Each card is a kind of "map" of your piece of cardboard and gives you a rule according to which you have to move your object. A general rule is that if there is a solid line dividing the card into two halves, the object must stay in the half in which it is. A dotted line is always a mirror. The object must go to the mirror image of its present position in the mirror. A "dot" like this o means a point symmetry about that point. The object must go, in a straight line past the "dot", to the other side of the "dot" which is as far from the "dot" as its original position was. The moves with arrows always mean that the object must be moved through two spaces, in the direction of the arrows. The KN moves are as knight's moves in chess, but the object must stay in the half in which it is before you move it. Use a ruler for cards C4 and D3!

Here is what you can do with the cards:
(a) Choose one half of the games board, top or bottom, left or right. Then find seven cards, which move the object around using their rules one after the other but so that it never leaves the half you have chosen.
(b) Choose any one of the cards. Move your object from any starting point to its finish as demanded by the card. Put another object at the same starting point. Now find three cards such that if you move your second object, following the rules of the cards you have chosen, at the end of your third move your second object will finish in the same space as your first object did. Then do it again with three more cards, but do not use any cards already used! Try to find five lots of three cards, each set of three bringing your object from the starting space to the same finishing space of your chosen first card.
(c) Here is a real brain-teaser:

Choose your first card as before. This is your first chosen card. Now place the remaining fifteen cards in a circular track, so that as you go round in the clockwise sense, then any three cards placed like this:
first card, second card, fifth card
will always take an object from the starting space of your first chosen card to the finishing space of your first chosen card.
Start your circle with four cards which are such that no three of them one after the other will make an object do what your first chosen card makes your object do in one move. If you can solve this problem, then you are ready to travel to Ruritania!


[^0]:    "It's quite correct, I've checked it!" said Alice to Bruce, who was still trying to work it out. Bruce was not aware of the sales person trying to hand him the goods he had purchased, while holding out his hands for the money!
    "I am sorry", said Bruce "I was just trying to work out what I owed you"

